Reconstruction of the hardening depth profile of steel rods

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Depth profilometry is a very important thermal wave inverse problem where the thermal diffusivity profile is reconstructed from the measurement of the surface temperature. Thermal diffusivity is a transport property which depends on the microstructure of the material and therefore can be used to identify changes that take place as a result of bulk modification processes, such as case hardening.

Microhardness testing is the most common technique to analyze the case depth in carburized and hardened steels. However, it is a destructive, time-consuming and expensive process. Since an anticorrelation between thermal diffusivity and hardness has been established, photothermal techniques provide a non-destructive method for monitoring hardness (indirectly). Actually, in the last decade many works devoted to the reconstruction of the thermal diffusivity depth profile from the measurement of the surface temperature have been published. However, all these works have been performed on flat surfaces. In this communication we present a method to reconstruct the thermal diffusivity depth profile of rods. We first develop the direct method which consists in calculating the surface temperature of a rod with in-depth varying thermal properties. It is simulated as formed by many concentric layers with different thermal properties. Then we use an inverse procedure to reconstruct the profile of the thermal properties from the knowledge of the surface temperature. Finally, infrared thermography is used to record the surface temperature behaviour of hardened steel rods. The reconstructed thermal diffusivity profile is compared with the hardness profile obtained by Vickers indentation test.

In figure 1 we show the cross-section of an infinite and opaque multilayer cylinder which is illuminated by a flat light beam of intensity I_o, modulated at a frequency f. The cylinder is made of N layers of different thicknesses and properties. The thermal properties of layer i are labelled by subindex i, and its outer and inner radii by a_i and a_{i+1} respectively. The thermal quadrupole method [1] is used to obtain the amplitude of the oscillating temperature at the surface in an elegant manner [2]:

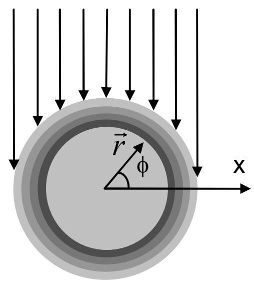

Fig. 1. *Geometry*

$$T(a_1,\phi)=\frac{I_o}{2}\sum_{m=-\infty}^{\infty}\frac{\left|\cos(m\pi/2)\right|}{\pi\left(1-m^2\right)}\frac{A'_m}{C'_m+A'_m\,h}e^{im\phi}\,, \tag{1}$$

where the frequency dependent coefficients A'_m and C'_m are obtained from the following matrix product

$$\begin{pmatrix} A'_m & B'_m \\ C'_m & D'_m \end{pmatrix}=\prod_{i=1}^{N}\begin{pmatrix} A_{mi} & B_{mi} \\ C_{mi} & D_{mi} \end{pmatrix}, \qquad m=-\infty,\ldots,0,\ldots,+\infty\,, \tag{2}$$

with

$$A_{mi}=\left[H'_{mi}(q_ia_{i+1})J_{mi}(q_ia_i)-J'_{mi}(q_ia_{i+1})H_{mi}(q_ia_i)\right]/E_{mi}\,,$$

$$B_{mi}=\left[J_{mi}(q_ia_{i+1})H_{mi}(q_ia_i)-H_{mi}(q_ia_{i+1})J_{mi}(q_ia_i)\right]/E_{mi}K_iq_i\,,$$

$$C_{mi}=K_iq_i\left[H'_{mi}(q_ia_{i+1})J'_{mi}(q_ia_i)-J'_{mi}(q_ia_{i+1})H'_{mi}(q_ia_i)\right]/E_{mi}\,,$$

$$D_{mi}=\left[J_{mi}(q_ia_{i+1})H'_{mi}(q_ia_i)-H_{mi}(q_ia_{i+1})J'_{mi}(q_ia_i)\right]/E_{mi}$$

and $E_{mi}=H'_{mi}(q_ia_{i+1})J_{mi}(q_ia_{i+1})-H_{mi}(q_ia_{i+1})J'_{mi}(q_ia_{i+1})$.

Here K is the thermal conductivity, $q = (1+i)/\mu$ is the thermal wave vector, being $\mu = (D/\pi f)^{0.5}$ the thermal diffusion length and D the thermal diffusivity. On the other hand, J_m, H_m, J'_m and H'_m are the m-th order of the Bessel and Hankel functions and their derivatives respectively.

In figure 2 we show the frequency dependence of the amplitude and phase of the surface temperature at the north pole ($\phi = \pi/2$) of a cylinder of radius 1 mm, whose thermal conductivity varies linearly from K = 10 $Wm^{-1}K^{-1}$ at the centre to K = 5 $Wm^{-1}K^{-1}$ at the surface ($K = 10 - 5\times10^3r$), while the heat capacity $\rho c = 2.5\times10^6$ S.I. remains constant. In all the simulations the temperature is normalized to the one of a homogeneous cylinder of the same radius and heat capacity but with a constant thermal conductivity K = 10 $Wm^{-1}K^{-1}$. The continuous line is the almost exact calculation, which is performed by reproducing the linear profile with 100 layers of the same thickness; the dashed-dotted line represents a 4-layer cylinder, the dashed line represents a 6-layer cylinder and the dotted line represents a 10-layer cylinder. As can be seen, results differ at high frequencies and a high number of layers would be required to simulate the linear profile accurately.

In order to reduce the number of layers needed to reproduce the linear profile we use layers of different thickness, thinner outside and thicker inside the cylinder. In figure 3 we show the amplitude and phase of the normalized surface temperature for the same cylinder of figure 2, but using layers whose thicknesses are multiple of the outer one. In this case with only 6 layers the exact behaviour is perfectly reproduced.

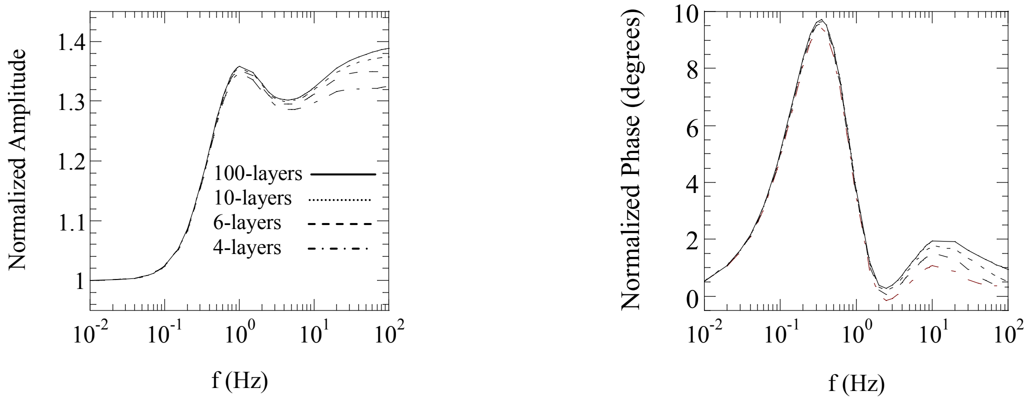

Fig. 2. *Amplitude and phase of the surface temperature at the north pole of a rod with a linearly varying in-depth thermal conductivity. Layers of equal thickness are used in the calculations.*

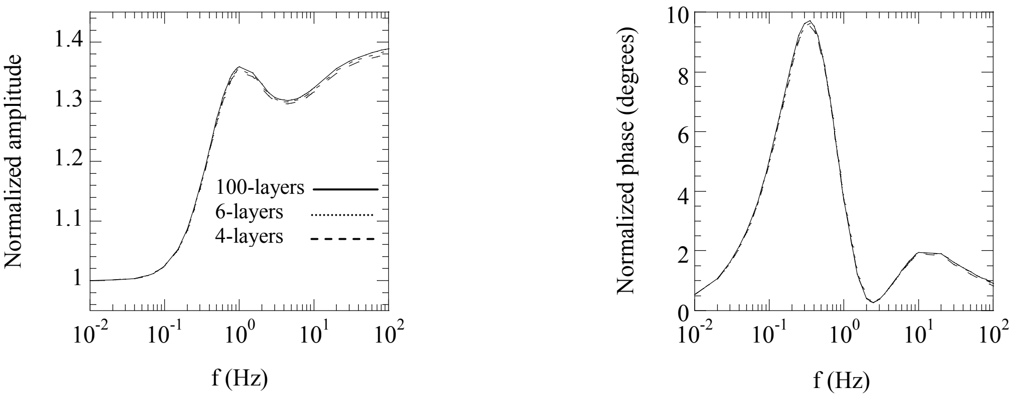

Fig. 3. *The same as in figure 2, but using layers of different thickness.*

We have measured a set of hardened steel rods with radii varying in the range from 1 mm to 5 mm. The temperature of the sample has been measured using two different infrared radiometric setups. For measurements in the frequency domain we have used an acousto-optically modulated Ar^+ laser as heating source together with a MCT detector with an active area of 1 mm^2 to capture the infrared emission from the sample and a lock-in amplifier. For measurements in the time regime we have used a 3kJ flash lamp as heating source, while the infrared emission is recorded and processed by a fast infrared camera (up to 7,000 frames/s).

To extract information about the thermal conductivity profile we have used a nonlinear least squares fitting procedure. We have defined a residual function (g) of the thermal conductivity as follows:

$$g(K) = \frac{1}{2}\sum_{j=1}^{N}\left|T_{theory}\left(K, f_j\right) - T_{measured}(f_j)\right|^2 , \qquad (3)$$

where $T_{measured}$ is the experimental value of the surface temperature (amplitude or phase) at a frequency f_i and T_{theory} is the theoretical value at that frequency, calculated by means of Eq. (1). The sum runs over all N modulation frequencies of the experiment. Determining the thermal conductivity profile is reduced to finding the set of parameters that minimizes g, by using the Levenberg-Marquardt method that is a trust-region modification of the Gauss-Newton algorithm. This method combines the advantages of the Gauss-Newton method (high order of convergence) and the steepest descent method (large region of convergence).

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